

# MTH 511a - 2020: Lecture 8 - extra

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## Ratio-of-Uniforms - the region $C$

There was some good discussion on the RoU region  $C$ . Particularly, there was a discussion on the  $v$  axis and whether  $v$  can ever be entirely negative or entirely positive. At the time, I stated that if the support of  $f$  was entirely negative then  $v$  is probably entirely negative. **This is not true!** Below is a sketch of a proof that  $(0, 0)$  is always a boundary point of  $C$ . This implies that any box around  $C$  must be of the form  $[a_1, a_2] \times [b_1, b_2]$  such that  $a_2 > 0$  and  $0 \in [b_1, b_2]$ .

The proof below is courtesy Chirag Jindal and another attempt was made by Sanket Agarwal. Well done Chirag and Sanket!

Let  $\mathcal{X}$  be the support of  $f$ . Recall

$$C = \left\{ (u, v) : 0 \leq u \leq \sqrt{f\left(\frac{v}{u}\right)} \right\}.$$

For  $v = 0$ ,

$$u : 0 \leq u \leq \sqrt{f\left(\frac{v}{u}\right)} = \sqrt{f(0)}$$

- If  $0 \in \mathcal{X}$ , then  $f(0) > 0$ , then there exists a  $u' \in (0, \sqrt{f(0)})$  such that  $(u', 0) \in C$ .
- If  $0 \notin \mathcal{X}$ , then  $f(0) = 0$  implying  $u = 0$ . But since  $0/0$  is undefined, this argument is not straightforward.

Let  $(\epsilon, v) \in C$  such that  $\epsilon > 0$ . Set  $k = v/\epsilon$  so that  $(\epsilon, k\epsilon) \in C$ . Letting  $\epsilon \rightarrow 0$ , we get that  $(0, 0)$  is a boundary point of  $C$ .

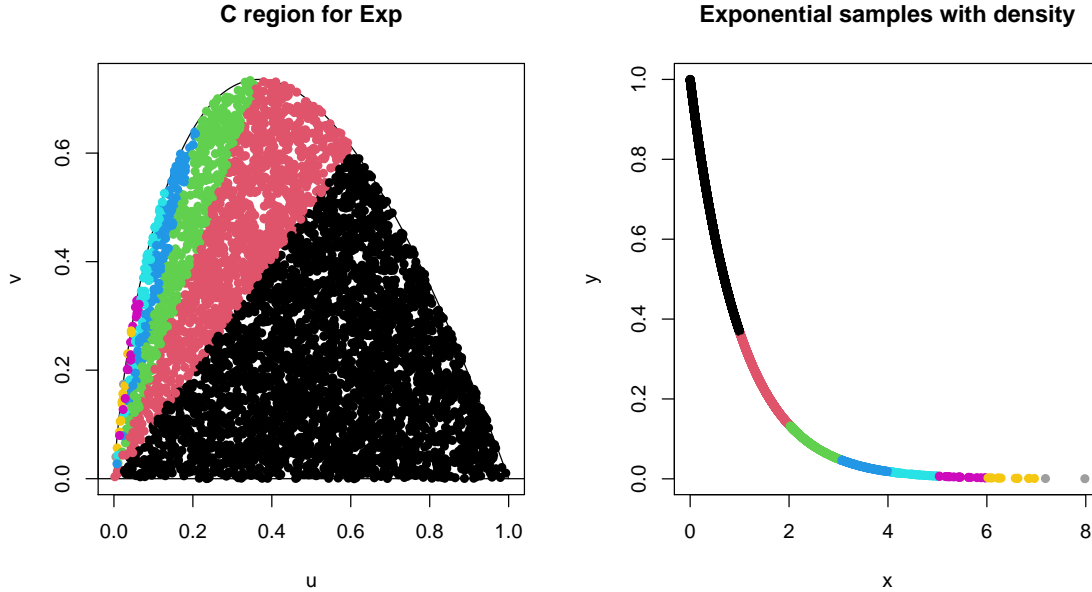
Example 1 (Exponential(1)).

$$f(x) = e^{-x} \quad x \geq 0$$

Here,

$$C = \{(u, v) : 0 \leq u \leq e^{-v/2u}\}.$$

The region  $C$  then is described by  $0 \leq u \leq 1$  and  $v \leq -2u \log u$ . Below is a plot of the region  $C$  and uniform samples from  $C$  color-coded with how they match with the support of  $f$ .



Example 2 (Normal(0,1)). The target density is:

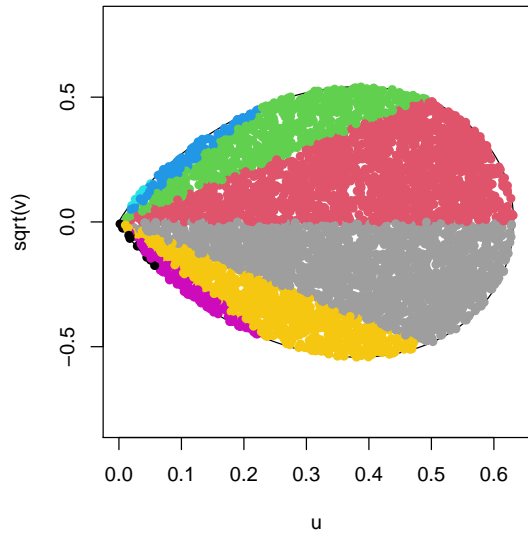
$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

The set  $C$  is

$$C = \left\{ (u, v) : 0 \leq u \leq \left( \frac{1}{2\pi} \right)^{1/4} e^{-v^2/4u^2} \right\}$$

The region  $C$  is defined by  $0 \leq u \leq 2\pi^{-1/4}$  and  $|v| \leq \sqrt{-4u^2 \log u + \frac{1}{4} \log 2\pi}$ . Below is a plot of the region  $C$  and uniform samples from  $C$  color-coded with how they match with the support of  $f$ .

**C region for  $N(0,1)$**



**Normal samples with density**

