Course Proposal: "Markov chain Monte Carlo"

Course description

A. Objective: The course will provide a theoretical foundation for constructing and studying Markov chain Monte Carlo (MCMC) algorithms, along with tools for analyzing MCMC output. Special focus will be given on rates of convergence of a Markov chain and comparing different MCMC algorithms. The course will primarily focus on discrete-time Markov chains on general state spaces. The objective is to equip students with the tools to develop, study, and implement an MCMC algorithm for any given problem.

B. Contents

S.No.	Broad Title	Topics	No. of lectures
1.	Motivation and definitions	a) Monte Carlo Integration, Bayesian estimation, Generalized linear Mixed Models b) Probability measure c) Markov kernels d) Detailed balance, Invariance, irreducibility, aperiodicity Elementary examples	4
2.	Constructing MCMC algorithms	a) Using reversibility: Metropolis-Hastings, Barker's kernels b) Combining kernels	2
3.	Metropolis-Hastings	a) Algorithm's definition b) Markov kernel and proof of detailed balance c) Different proposal distributions d) Example of MH and Barker's algorithms in real problems	3
4.	Gibbs Samplers	a) Component-at-a-time MH b) Deterministic Scan Gibbs samplers c) Random Scan Gibbs samplers d) Example of Gibbs Samplers in real models	2
5.	Markov chain Convergence and CLT	a) Total variation distance b) Harris recurrence and ergodicity c) Expression of asymptotic variance d) Proof of CLT under general conditions e) Sufficient conditions for CLT	5
6.	Rates of convergence of Markov chain	a) Three rates of convergence b) Special focus on uniformly ergodic and discrete state space Markov chains c) Couplings d) Drift and minorizations e) Geometric ergodicity using couplings	10

		f) Spectral properties and Cheeger inequality g) Examples	
7.	Comparing Markov chains	a) First comparison theorem b) Relation between various Gibbs samplers c) Peskun's Ordering	2
8.	Estimation using MCMC	a) Estimating expectations b) Quantiles c) Joint CLT of quantiles and expectations	2
9.	Variance estimation	a) Spectral methods b) Initial sequence estimators c) Batch means estimators d) Strong consistency of estimators	3
10.	Stopping criterion for MCMC	a) Sequential stopping rules b) Effective sample size c) Gelman-Rubin diagnostic d) Relation between Gelman-Rubin and ESS	4
11.	Implementing MCMC algorithms in R	a) Writing MCMC code b) Inbuilt packages in R c) Visualization tools	3
Total			**40**

C. Pre-requisites: MTH309A or equivalent, MTH431A or equivalent. Familiarity with Bayesian Analysis is preferred (MTH535A or equivalent), and consent of instructor.

D. Short summary: This course presents the theoretical and practical challenges of implementing a discrete-time general state space Markov chain Monte Carlo algorithm. Metropolis-Hastings, Gibbs samplers, and other component-wise algorithms are discussed in detail. The theoretical part of the course focuses on studying rates of convergence of Markov chains, and establishing the existence of a Markov chain central limit theorem. The practical challenges of implementing these algorithms, such as step-sizes, stopping criterion, output analysis, and implementation in statistical software are also discussed in detail.

Recommended Books and reference material

a) Meyn, Sean P., and Richard L. Tweedie. *Markov chains and stochastic stability*. Springer Science & Business Media, 2012.
b) Nummelin, Esa. *General irreducible Markov chains and non-negative operators*. Vol. 83. Cambridge University Press, 2004.
c) Brooks, Steve, Andrew Gelman, Galin Jones, and Xiao-Li Meng, eds. *Handbook of Markov chain Monte Carlo*. CRC press, 2011.
d) Robert, Christian, and George Casella. *Monte Carlo statistical methods*. Springer Science & Business Media, 2013.
e) Lindvall, Torgny. *Lectures on the coupling method*. Courier Corporation, 2002.

f) Roberts, Gareth O., and Jeffrey S. Rosenthal. "General state space Markov chains and MCMC algorithms." *Probability surveys* 1 (2004): 20-71.

g) Jones, Galin L. "On the Markov chain central limit theorem." *Probability surveys* 1, no. 299-320 (2004): 5-1.
h) Glynn, Peter W., and Ward Whitt. "The asymptotic validity of sequential stopping rules for stochastic simulations." *The Annals of Applied Probability* 2, no. 1 (1992): 180-198.
i) Roberts, Gareth O., and Jeffrey S. Rosenthal. "Optimal scaling for various Metropolis-Hastings algorithms." *Statistical Science*16, no. 4 (2001): 351-367.
j) Jarner, Søren Fiig, and Ernst Hansen. "Geometric ergodicity of Metropolis algorithms." *Stochastic Processes and their Applications* 85, no. 2 (2000): 341-361.

Other Departments/IDPs which may be interested in the course: Computer Science, Electrical Engineering, and Economics.

Proposing Instructor: Dootika Vats (dootika@iitk.ac.in)
Proposing Department: Department of Mathematics and Statistics