Methods of estimating Causal Effect

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August 31, 2020

Outline of the talk

- Specification of the problem.

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- Role of causal graphs, especially DAGs.
- Methods to control for confounding (observed or unobserved).

Notations

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Then potential outcomes will be denoted by collection of values Y^a for each possible a considered under study.

- We will consider only causal effects of treatments that we can imagine being randomized or manipulated in a hypothetical trial.

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- Causal effects of variables like age, race, gender etc. do not fit so cleanly in the potential outcome framework.

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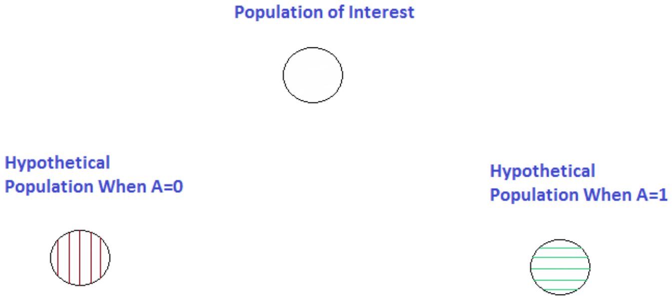
Do the outcomes on average differ if we treat whole population with treatment $A = 0$ versus $A = 1$?

Average Causal Effect



Difference of means $\mathbb{E}(Y^1) - \mathbb{E}(Y^0)$ will be the Average Causal Effect.

Subset versus Hypothetical

In practice, sub-populations might differ from population and what we can estimate from data is $\mathbb{E}(Y \big| A = 1) - \mathbb{E}(Y \big| A = 0)$

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which is generally not equal to $\mathbb{E}(Y^1) - \mathbb{E}(Y^0)$.

Assumptions for estimation

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- Positivity : For every set of covariates X, treatment assignment is such that $\mathbb{P}(A = a \big| X = x) > 0, \ \forall x$;

- Ignorability : For particular $X = x$, treatment assignment is independent from potential outcomes, i.e.,

$$(Y^0, Y^1) \perp\!\!\!\perp A \big| X$$

Observed Data and Potential Outcomes

Due to consistency, we have :

$$\mathbb{E}(Y \big| A = a, X = x) = \mathbb{E}(Y^a \big| A = a, X = x)$$

$$= \mathbb{E}(Y^a \big| X = x) \quad \text{by ignorability}$$

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Suppose X is categorical, then standardization gives :

$$\mathbb{E}(Y^a) = \sum_x \mathbb{E}(Y \big| A = a, X = x)\mathbb{P}(X = x) \tag{1}$$

Difficulties with Standardization

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- Stratification would lead to no data case for many combinations of levels of covariates.
- Some alternatives to standardization for estimating causal effects : matching, propensity score and inverse probability of treatment weighting.

Why Ignorability ?

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But why do we care about the mechanism ?

In many cases, 'sicker' patients are more likely to be treated or treated patients might be at higher risk of some bad outcome.

Within levels of covariates (e.g., people of same age, with same co-morbid conditions etc.), treatment assignment would not be dependent on potential outcomes and hence, through assignment mechanism, no bias would arise.

Identify Covariates

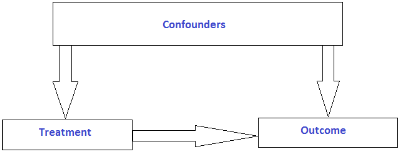

Figure: Real Situation

Identify Covariates

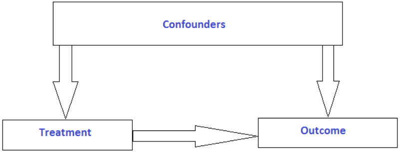

Figure: Real Situation

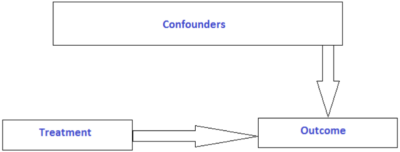

Figure: For Ignorability

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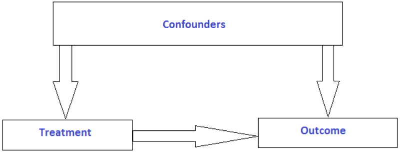

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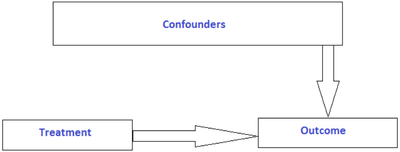


Figure: For Ignorability

If we clearly identify the variables we need to consider so as to achieve ignorability, then we can estimate average causal effect $\mathbb{E}(Y^1) - \mathbb{E}(Y^0)$ using equation (1).

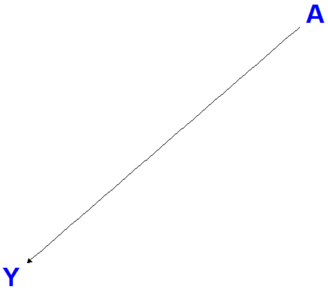
Causal Graphs

Causal Graphs will help us in determining the variables to control for.

Causal graphs encode assumptions about relationship among variables (i.e., which are independent, dependent and conditionally independent) along-with displaying explicit direction of effect from one to other variable.

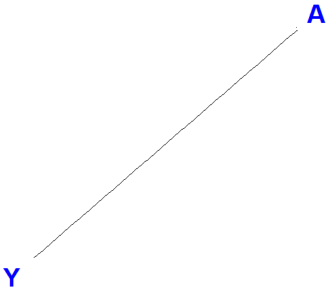
Directed vs. Undirected

Directed :



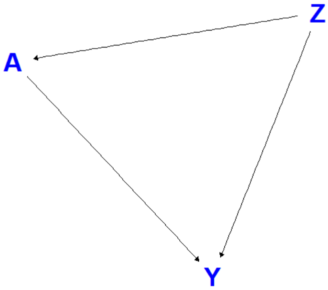
(a) A affects Y

Undirected Graph :

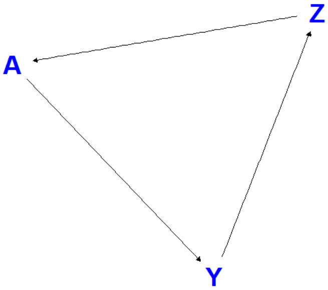


(b) A and Y are associated

Acyclic vs. Cyclic

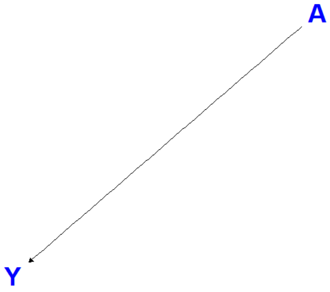


(a) Acyclic Graph



(b) Cyclic Graph

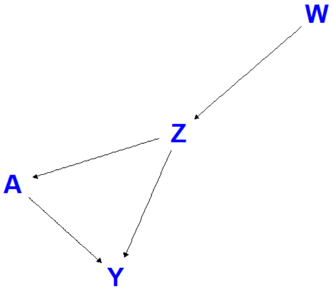
Terminology



A affects Y

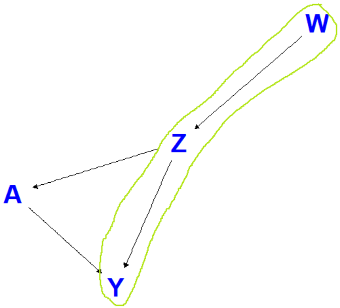
- Variables A and Y are known as nodes or vertices;
- Arrow form A to Y is an edge;
- Variables connected by an edge are adjacent.

Paths

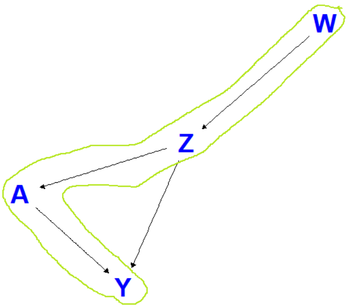


- A path is a way to get from vertex to another along edges;
- There are two paths from W to Y.

Paths

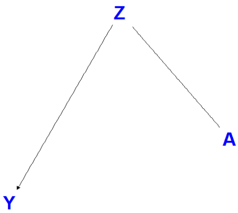


(a) $W \to Z \to Y$

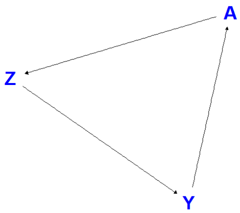


(b) $W \to Z \to A \to Y$

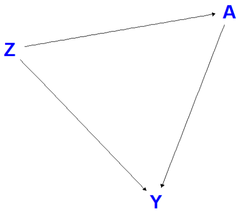
Directed Acyclic Graphs (DAGs)



(a) Undirected Acyclic

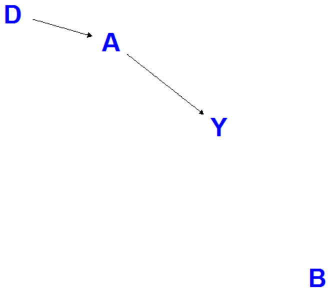


(b) Directed Cyclic



(c) DAG

Probabilistic interpretation of DAGs



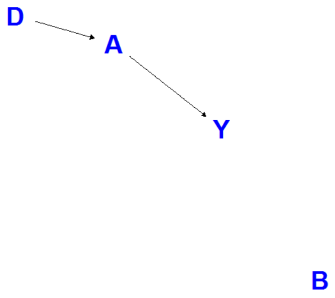
- $\mathbb{P}(B \big| A, D, Y) = \mathbb{P}(B)$
- $\mathbb{P}(Y \big| A, D, B) = \mathbb{P}(Y \big| A)$

$$\implies Y \perp\!\!\!\perp (D, B) \big| A$$

- $\mathbb{P}(D \big| A, B, Y) = \mathbb{P}(D \big| A)$

Example 1

Probabilistic interpretation of DAGs



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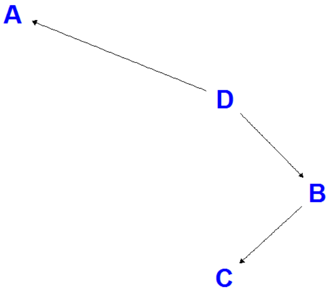
  $\implies Y \perp\!\!\!\perp (D, B) \big| A$
- $\mathbb{P}(D \big| A, B, Y) = \mathbb{P}(D \big| A)$
- No more independence ?

More examples on interpretation



Example 2

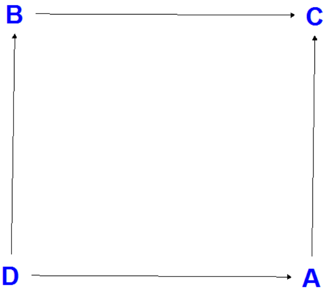
- $\mathbb{P}(D \big| A, C, B) = \mathbb{P}(D \big| A, B)$

  $\implies D \perp\!\!\!\perp C \big| B$

- $\mathbb{P}(A \big| C, D, B) = \mathbb{P}(A \big| D)$

  $\implies A \perp\!\!\!\perp (B, C) \big| D$

Examples Continued



Example 3

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- $\mathbb{P}(A \big| C, D, B) = \mathbb{P}(A \big| D)$

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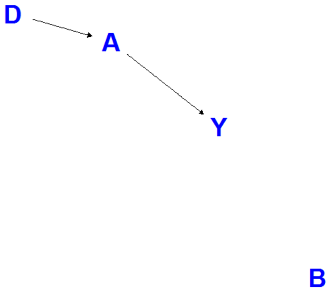
Joint Distribution

Suppose $X_1, X_2, ..., X_n$ are jointly distributed random variables, then by multiplication rule of conditional probability, we know that :

$$\mathbb{P}(X_1, X_2, X_3, ..., X_n) = \mathbb{P}(X_1) \times \mathbb{P}(X_2 \big| X_1) \times \mathbb{P}(X_3 \big| X_1, X_2) \times$$

$$... \times \mathbb{P}(X_n \big| X_1, X_2, X_3, ..., X_{n-1})$$

Decomposition Example 1



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$$\mathbb{P}(A, B, Y, D) =$$

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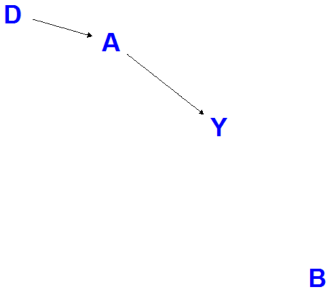
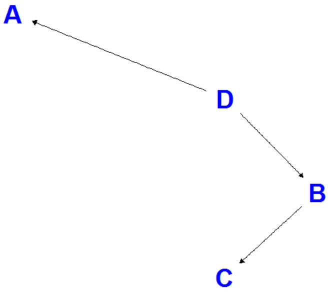


Example 1

$$\mathbb{P}(A, B, Y, D) =$$

$$\mathbb{P}(B) \times \mathbb{P}(D) \times \mathbb{P}(A \middle| D) \times \mathbb{P}(Y \middle| A)$$

Decomposition Example 2



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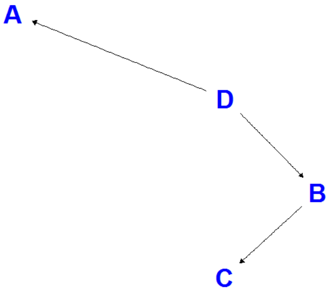
Example 2

Decomposition Example 2



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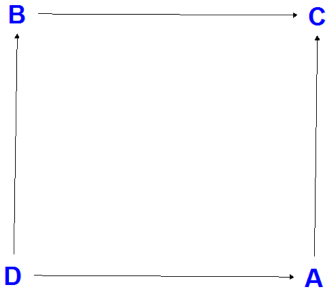
Example 2

Decomposition Example 3



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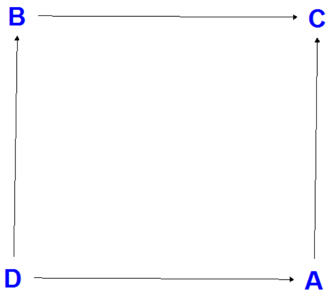

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DAGs and Distributions

For a given DAG, there is a unique decomposition of joint probability distribution.

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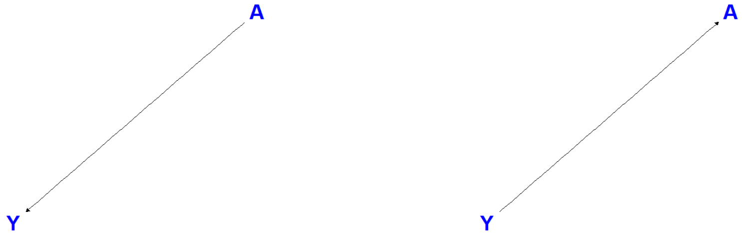
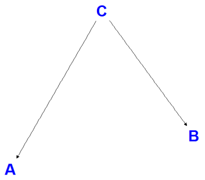
What about converse ?



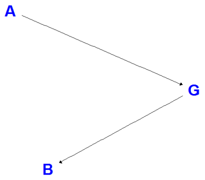
(a) A affects Y (b) Y affects A

Association through Paths

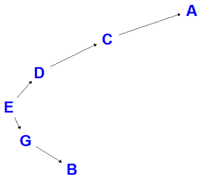
Nodes A and B are associated (via particular path) if :



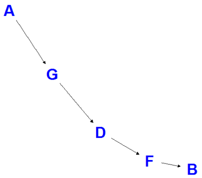
(a) Fork



(b) chain

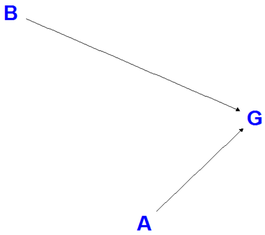


(c) Long Fork

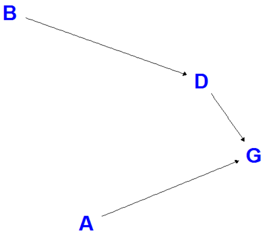


(d) Long Chain

Paths that do not induce Association

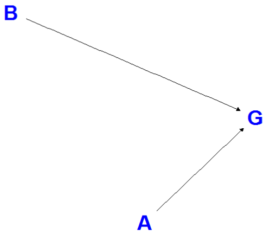


(a) Inverted Fork



(b) Inverted Fork

Paths that do not induce Association



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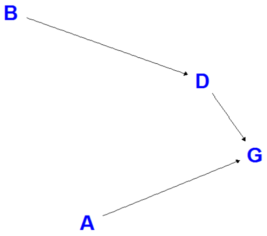


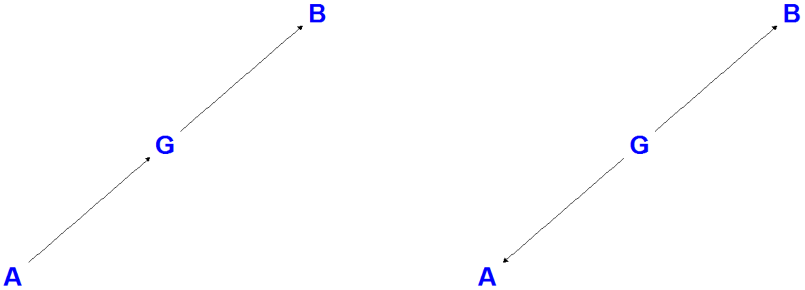
(b) Inverted Fork

Information flows from nodes A and B to G but collide and do not flow to each other which implies A and B are independent through the path.

G is known as collider

Blocking

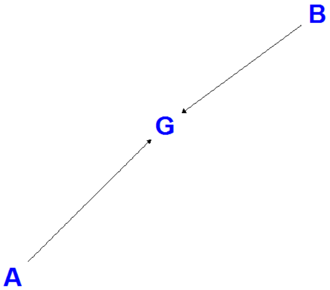
We can block the path by conditioning on nodes when nodes on the end are associated through this path.



(a) (b)

Blocking is not always a solution !

A and B are not associated marginally via this path.



G is a collider

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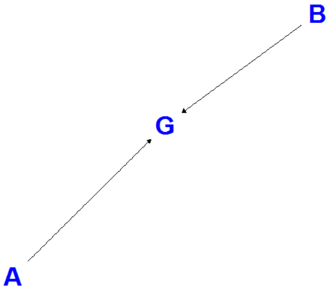
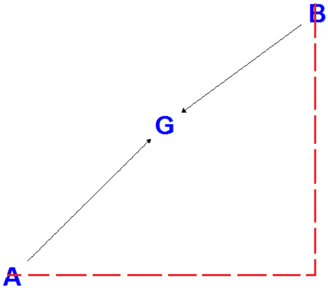


G is a collider

But, conditioning on G induces association between A and B.



Conditioned on G

path d-separated

A path p is d-separated by a set of nodes S if :

path d-separated

A path p is d-separated by a set of nodes S if :

- p contains a chain $(D \rightarrow E \rightarrow F)$ and the middle part of chain is in S, or;

path d-separated

A path p is d-separated by a set of nodes S if :

- p contains a chain ($D \rightarrow E \rightarrow F$) and the middle part of chain is in S, or;
- p contains a fork ($D \leftarrow E \rightarrow F$) and the middle part of fork is in S, or;

path d-separated

A path p is d-separated by a set of nodes S if :

- p contains a chain $(D \rightarrow E \rightarrow F)$ and the middle part of chain is in S, or;
- p contains a fork $(D \leftarrow E \rightarrow F)$ and the middle part of fork is in S, or;
- p contains an inverted fork $(D \rightarrow E \leftarrow F)$ and middle part is not in S, nor any of its descendants are in S.

d-separation

Two nodes, A and B are d-separated by a set of nodes S if it blocks every path from A to B.

Above implies, $A \perp\!\!\!\perp B \big| S$

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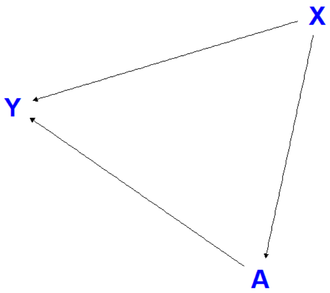
Ignorability satisfied ?

$(Y^0, Y^1) \perp\!\!\!\perp A \big| X$

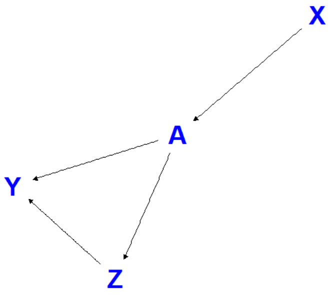
Frontdoor Path

Frontdoor path from A (treatment or exposure) to Y (outcome) is the path that begins with an arrow emanating out of A.



- $A \rightarrow Y$ is a frontdoor path from A to Y.
- Here, A affects Y directly.

Effect through Frontdoor



- $A \rightarrow Z \rightarrow Y$ is a frontdoor path from A to Y.
- Here, A affects Y indirectly through its effect on Z.

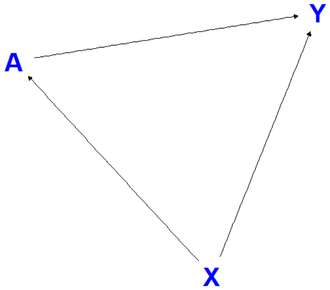
We care about how Y is affected by manipulating A !

Controlling for Z would be controlling for some portion of affect of treatment which is not desired.

Backdoor paths

Causal Mediation Analysis focuses on quantifying effect of treatment to outcome through frontdoor paths.

Backdoor paths from A to Y are paths from A to Y that travel through arrows going into A.

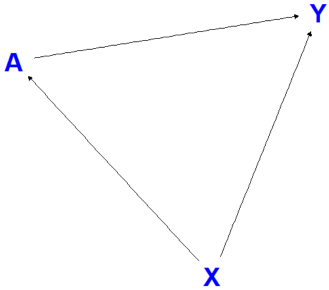


$A \leftarrow X \rightarrow Y$ is backdoor path from A to Y

Backdoor paths

Causal Mediation Analysis focuses on quantifying effect of treatment to outcome through frontdoor paths.

Backdoor paths from A to Y are paths from A to Y that travel through arrows going into A.



Backdoor paths confound the relationship between A and Y.

$A \leftarrow X \rightarrow Y$ is backdoor path from A to Y

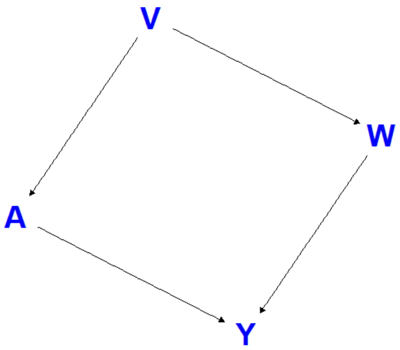
Criterion

A set of variables X is sufficient to control for confounding if:

- it blocks all backdoor paths from treatment to the outcome.
- it does not include any descendants of the treatment.

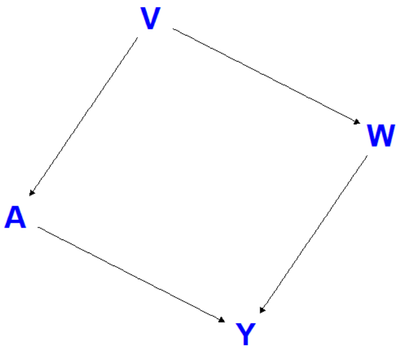
This is the backdoor path criterion.

Example 1



$A \leftarrow V \rightarrow W \rightarrow Y$ is backdoor path from A to Y

Example 1



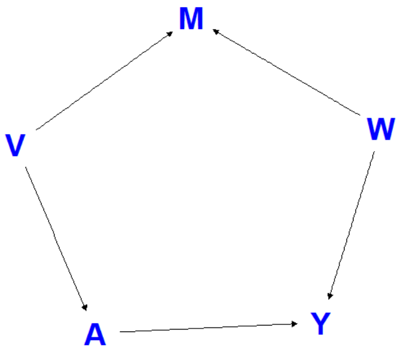
$A \leftarrow V \rightarrow W \rightarrow Y$ is backdoor path from A to Y

Not blocked by a collider.

Sets of variables:

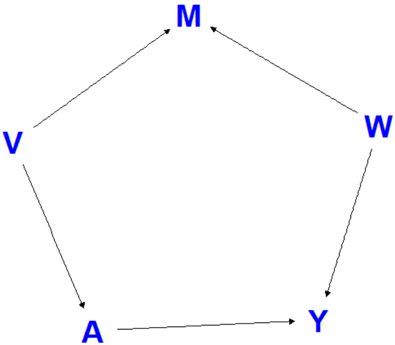
- {V};
- {W};
- {V,W}

Example 2



$A \leftarrow V \rightarrow M \rightarrow W \rightarrow Y$ is backdoor path from A to Y

Example 2

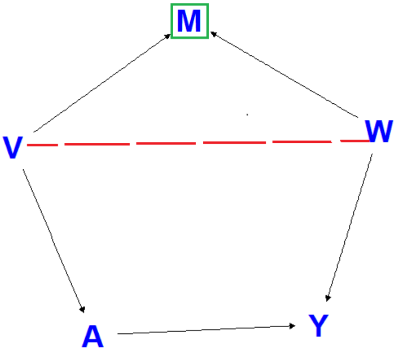


$A \leftarrow V \rightarrow M \rightarrow W \rightarrow Y$ is backdoor path from A to Y

Backdoor Path is blocked by a collider M.

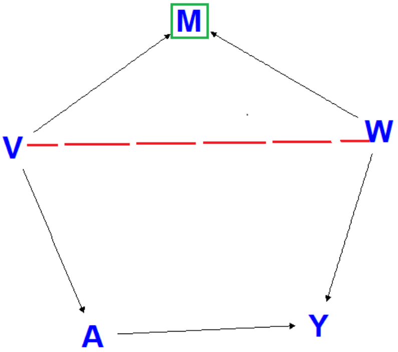
No need to control any variable.

Example 2 Continued



$A \leftarrow V \rightarrow M \rightarrow W \rightarrow Y$ is backdoor path from A to Y

Example 2 Continued

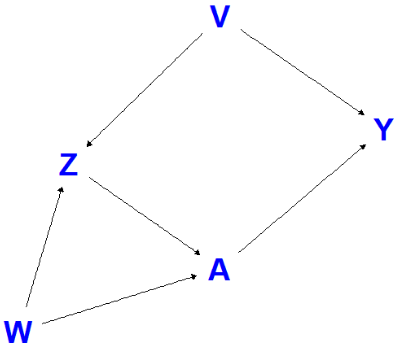


$A \leftarrow V \rightarrow M \rightarrow W \rightarrow Y$ is backdoor path from A to Y

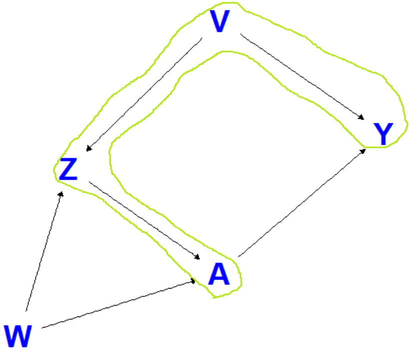
Sets of variables:

- {}, {V}, {W};
- {V,W}, {M,V},{M,W};
- {M,V,W}

Example 3



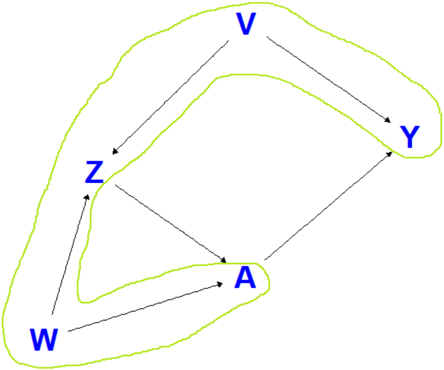
Example 3



First Backdoor Path:

$A \leftarrow Z \leftarrow V \rightarrow Y$

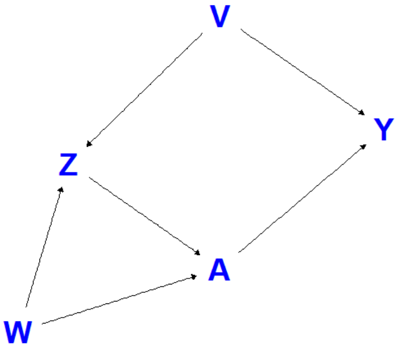
Example 3



Second Backdoor Path:

$A \leftarrow W \rightarrow Z \leftarrow V \rightarrow Y$

Example 3 Continued

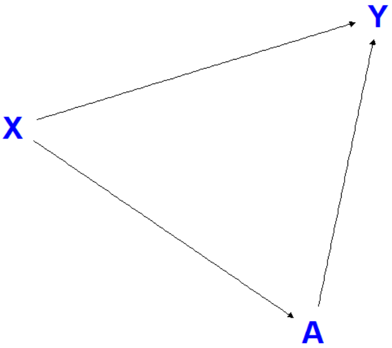


Sets of variables:

- {V};
- {V,Z},{V,W},{Z,W};
- {V,Z,W}.

Not included {Z} or {W}

How to Control ?



If X is observed:

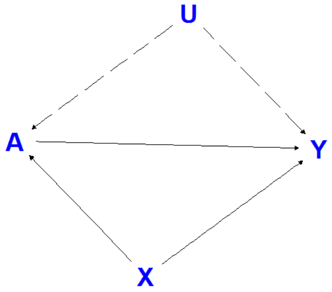
- Matching;
- Propensity Score Matching;
- Inverse Probability Treatment Weighting.

Unmeasured Confounding

- Suppose U is unobserved variable that affects A and Y, then we have unmeasured confounding.
- Ignorability assumption is violated.
- Cannot control for confounders if we do not observe them all by using previous methods.

Unmeasured Confounding

- Suppose U is unobserved variable that affects A and Y, then we have unmeasured confounding.
- Ignorability assumption is violated.
- Cannot control for confounders if we do not observe them all by using previous methods.
- Instrumental Variables (IV) is an alternative causal inference method that does not rely on ignorability assumption.



Literature

- Textbooks : Judea 2010; Spirtes, C. N. Glymour, et al. 2000; Pearl et al. 2016
- Multiple treatment effects : Lopez, Gutman, et al. 2017
- Incomplete data : Gelman and Meng 2004
- Critics on subject received : Humphreys and Freedman 1996; Spirtes, C. Glymour, et al. 1997
- Comparison of observational and experimental : Rosenbaum 2017
- Causal Graphs : Scheines 1997; Greenland and Pearl 2014

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THANK
YOU